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PROCEDURE FOR CALCULATING A CONDUCTIVE HEAT EXCHANGER

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In a number of devices a fluid is heated as it flows through a porous wall or a metal wall with a high thermal conductivity which has channels containing the flowing fluid. There naturally arise problems of finding the temperature distribution in the material of such a wall heat exchanger, the temperature change of the flowing fluid, and the amount of heat transferred for given geometric parameters of the wall heat exchanger and given values of the initial temperature and the flow rate of the fluid. To simplify the problems we make the following assumptions: 1) the thermal and physical properties and the heat-transfer characteristics are constants; 2) end effects related to the finite dimensions of the wall heat exchanger can be neglected, i.e., we assume an infinite porous wall; 3) heat transfer through the skeleton of the wall is by conduction only in the direction normal to the boundary surfaces, i.e., the temperature gradient in the transverse direction can be neglected. This condition holds when the temperature difference between the central part of individual elements of the skeleton and the channel surface is small in comparison with the temperature difference between the channel surface and the moving fluid, i.e., the thermal resistance to conduction through the heat-exchanger skeleton and the thermal resistance to heat transfer to the moving fluid are controlling. This assumption will be satisfied sufficiently rigorously if the Biot number $Bi = \alpha h/\lambda$ determined in the heat-exchanger skeleton is smaller than unity. [1]. It is easy to verify that for metal heat exchangers this assumption is satisfied over reasonable variations of the heat-transfer coefficient α, the thermal conductivity λ, and a characteristic internal dimension h — the distance between centers of the openings; 4) heat conduction in the fluid can be neglected; this assumption is obvious for turbulent flow, but even for laminar flow, taking account of the fact that the thermal conductivity of a fluid is an order of magnitude smaller than that of the heat-exchanger material, heat conduction in the fluid can be neglected.

Thus, in our heat-exchanger model we assume that heat is transferred by conduction through the skeleton of the material, and by convection to the fluid. Then the system of equations for the temperature distribution (a one-dimensional problem for the material of the device) can be formulated in the following way for a 1-m² cross section of the heat exchanger (Fig. 1).

The amount of heat transferred to the flowing fluid in a part dx during a time $d\tau$ is [1]

$$dQ = \lambda f \frac{d^2 t_M}{dx^2} dx d\tau, \tag{1}$$

but on the other hand this heat can be determined from the heat-transfer equation

$$dQ = \alpha P (t_M - t_F) dx d\tau. \tag{2}$$

Finally, the amount of heat going into heating of the fluid is

$$dQ = \rho (1 - f) u c_p \frac{dt_F}{dx} dx d\tau. \tag{3}$$

By equating (1), (2), and (3), we obtain the following system of equations:

$$\lambda f \frac{d^2 t_M}{dx^2} = \alpha P (t_M - t_F), \tag{4}$$

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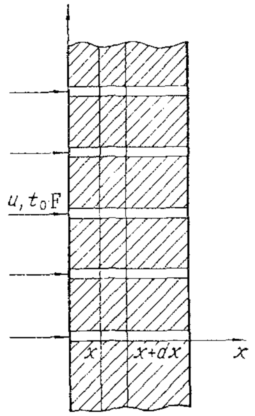

Fig. 1. Schematic diagram of heat-exchanger wall.

$$\rho(1-f)uc_p\frac{dt_F}{dx}=\alpha P(t_M-t_F).$$

By introducing the dimensionless argument $\xi = x/\delta$ and the notation

$$\mathrm{Bi}_\delta=\frac{\alpha P\delta^2}{\lambda f},\quad g=\frac{\rho(1-f)uc_p\delta}{\lambda f},\tag{5}$$

we obtain the system

$$\frac{d^2t_M}{d\xi^2}=\mathrm{Bi}_\delta(t_M-t_F),\quad \frac{dt_F}{d\xi}=\frac{\mathrm{Bi}_\delta}{g}(t_M-t_F),\tag{6}$$

which we solve for two variants of the boundary conditions.

First Variant. The temperatures of the end surfaces of the heat exchanger and the inlet temperature of the fluid are specified, i.e., $t_M|_{\xi=0} = t_{0M}$, $t_M|_{\xi=1} = t_{1M}$, $t_F|_{\xi=0} = t_{0F}$.

The temperatures of the end surfaces of the heat exchanger are maintained at the specified values by external heat transfer. This can be accomplished by using a shield which transmits radiant energy to the end surface. The temperature of the surface signals for a change in the amount of heat liberated by the shield.

Thus, a very simple automated system solves the problem of maintaining the end surfaces at specified temperatures.

By introducing the dimensionless quantities

$$\Theta_M=\frac{t_M-t_{0M}}{t_{0F}-t_{0M}},\quad \Theta_F=\frac{t_F-t_{0M}}{t_{0F}-t_{0M}},\tag{7}$$

we obtain the following problem for analysis:

$$\frac{d^2\Theta_M}{d\xi^2}=\mathrm{Bi}_\delta(\Theta_M-\Theta_F),\quad \frac{d\Theta_F}{d\xi}=\frac{\mathrm{Bi}_\delta}{g}(\Theta_M-\Theta_F),\tag{8}$$

$$\Theta_M(0)=0,\ \Theta_M(1)=\frac{t_{1M}-t_{0M}}{t_{0F}-t_{0M}}=\Theta_0,\tag{9}$$

$$\Theta_F(0)=1.\tag{10}$$

By eliminating Θ_F from system (8) we obtain the following linear differential equation for Θ_M:

$$\frac{d^3\Theta_M}{d\xi^3}+\frac{\mathrm{Bi}_\delta}{g}\frac{d^2\Theta_M}{d\xi^2}-\mathrm{Bi}_\delta\frac{d\Theta_M}{d\xi}=0\tag{11}$$

with boundary conditions (9). We obtain another boundary condition from the first of Eqs. (8):

$$\left.\frac{d^2\Theta_M}{d\xi^2}\right|_{\xi=0}=-\mathrm{Bi}_\delta.\tag{12}$$

Taking account of (9) and (12), the solution of Eq. (11) has the form [2]

$$\Theta_M = C_1(\exp n\xi - 1) - C_2[1 - \exp(-m\xi)], \tag{13}$$

where

$$m = \frac{Bi_\delta}{2g} + \sqrt{\left(\frac{Bi_\delta}{2g}\right)^2 + Bi_\delta}\ ; \tag{14}$$

$$n = -\frac{Bi_\delta}{2g} + \sqrt{\left(\frac{Bi_\delta}{2g}\right)^2 + Bi_\delta}\ ; \tag{15}$$

$$C_1 = \frac{Bi_\delta[1 - \exp(-m)] - \Theta_0 m^2}{n^2[\exp(-m) - 1] + m^2(1 - \exp n)}\ ; \tag{16}$$

$$C_2 = \frac{Bi_\delta(\exp n - 1) + \Theta_0 n^2}{n^2[\exp(-m) - 1] + m^2(1 - \exp n)}\ . \tag{17}$$

Using the first of Eqs. (8), it is easy to find the temperature of the flowing fluid

$$\Theta_F = 1 + \frac{1}{g}[nC_1(\exp n\xi - 1) + mC_2(1 - \exp(-m\xi)] \tag{18}$$

and the dimensionless form of the amount of heat transferred

$$\bar{Q} = \frac{Q\delta}{\lambda f(t_{0F} - t_{0M})} = C_1 n(\exp n - 1) + C_2 m[1 - \exp(-m)] \tag{19}$$

or, as is easy to see,

$$\bar{Q} = g[\Theta_F(1) - 1]. \tag{20}$$

Analysis of Eqs. (5) shows that for the most characteristic geometric and operating parameters of heat exchangers the Biot number Bi_δ may vary over rather wide limits from 0 to 10,000, whereas the value of Bi_δ/g is limited to the range 0.1-0.6. Therefore it is of interest to investigate the solution obtained for the limiting values of Bi_δ.

Suppose $Bi_\delta \to 0$ (in practice $Bi_\delta < 0.5$). In this case the change in metal temperature is given by $\Theta_M = \Theta_0\xi$, and the fluid temperature is practically unchanged: $\Theta_F = 1$, i.e., in this case it can be said that the heat exchanger is not working. This results from the fact that as $Bi_\delta \to 0$ the heat-transfer surface (the inner surface of the channels $P\delta$) is also decreased, while the relative metal surface f is increased. Heat transfer between the coolant and metal turns out to be negligible in comparison with heat transfer by conduction. The metal temperature varies linearly, as it would if there were no flowing fluid.

Suppose $Bi_\delta \to \infty$ (in practice $Bi_\delta > 100$). In this case the diameter of the channels is relatively large, which means that the heat-transfer surface is also large and the metal volume is small. Since Bi_δ/g varies only from 0.1 to 0.6, the coolant flow rate will be large. The metal takes on the temperature of the fluid practically instantaneously, and heat transfer occurs only at the boundaries where $\xi = 0$ and $\xi = 1$. The inner part of the heat exchanger in fact is inoperative.

Figure 2a shows the most characteristic temperature distributions in metal and fluid for $Bi_\delta = 10$ and Bi_δ/g equal to 0.1, 0.2, 0.3, 0.4, and 0.5, with the temperatures of the heat-exchangers walls maintained at $t_{0M} = t_{1M}$, i.e., $\Theta_0 = 0$. It is clear from the figure that even for a very small increase in Bi_δ/g, i.e., a decrease in the coolant flow rate, the difference between the initial and final coolant temperatures increases appreciably, and calculations show that heat transfer increases somewhat. In turn the metal temperature hardly changes for a change in Bi_δ/g.

<u>Second Variant.</u> The temperature of the material on one of the heat-exchanger surfaces is specified $t_M|_{\xi_1} = t_{1M}$, and the second surface is thermally insulated, i.e.,

$$\left.\frac{dt_M}{d\xi}\right|_{\xi=0} = 0,$$

By introducing the dimensionless temperatures

$$T_M = \frac{t_M - t_{0F}}{t_{1M} - t_{0F}}\ ,\ T_F = \frac{t_F - t_{0V}}{t_{1M} - t_{0F}}\ , \tag{21}$$

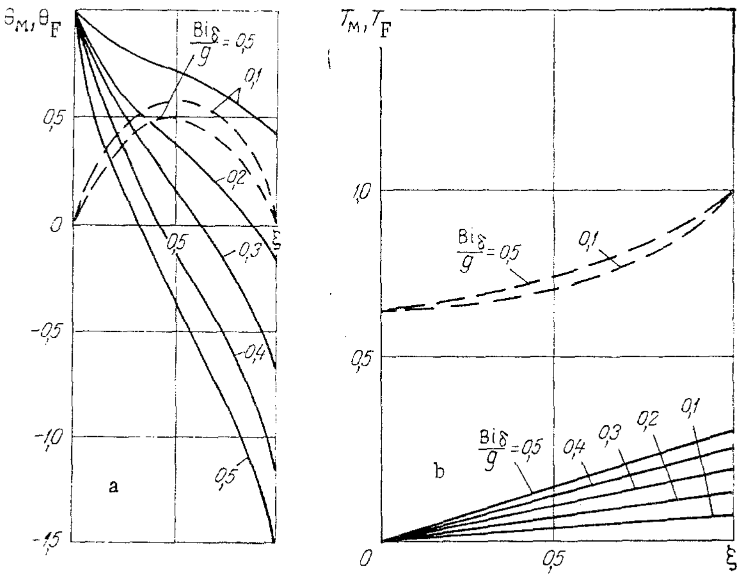

Fig. 2. a) Temperatures of metal Θ_M (open curves) and fluid Θ_F (solid curves) for $Bi_\delta = 10$; b) temperatures of metal T_M (open curves) and fluid T_F (solid curves) for $Bi_\delta = 1$.

we obtain the problem

$$\frac{d^2T_M}{d\xi^2} = Bi_\delta (T_M - T_F), \quad \frac{dT_F}{d\xi} = \frac{Bi_\delta}{g}(T_M - T_F), \tag{22}$$

$$\left.\frac{dT_M}{d\xi}\right|_{\xi=0} = 0, \quad T_M|_{\xi=1} = 1, \tag{23}$$

$$T_F|_{\xi=0} = 0, \tag{24}$$

whose solution is

$$T_M = \frac{n\exp(-m\xi) + m\exp(n\xi)}{n\exp(-m) + m\exp n}, \tag{25}$$

$$T_F = \frac{Bi_\delta}{g}\,\frac{\exp n\xi - \exp(-m\xi)}{n\exp(-m) + m\exp n}. \tag{26}$$

In dimensionless form the amount of heat transferred to the coolant is

$$\bar{Q} = \frac{Q\delta}{\lambda f(t_{1M} - t_{0F})} = Bi_\delta \frac{\exp n - \exp(-m)}{n\exp(-m) + m\exp n} = gT_F(1). \tag{27}$$

Analysis of the formulas obtained shows that the fluid temperature at the heat-exchanger outlet increases with decreasing Bi_δ. If $Bi_\delta \to \infty$ (in practice $Bi_\delta > 10$), the heat exchanger does not operate, since the fluid temperature does not change, and the metal takes on the temperature of the fluid instantaneously. The most characteristic temperature distributions for this variant are shown in Fig. 2b. Here $Bi_\delta = 1$, and Bi_δ/g is varied from 0.1 to 0.5. As for the first variant, we see that while the coolant temperature varies with a change in Bi_δ/g, the change in the metal temperature is quite negligible.

The analysis presented shows that for $Bi_\delta < 1$ heat exchangers of the second type, i.e., those with one insulated surface, are more efficient than those of the first type, while for $100 > Bi_\delta > 1$ heat exchangers of the first type are considerably more efficient than those of the second type, since for the values of Bi_δ indicated, heat exchangers of the second type are practically inoperative. This important fact should be taken into account in the design and calculation of heat exchangers.

NOTATION

ρ, fluid density; c_p, specific heat; u, fluid velocity; λ, thermal conductivity of metal; x, dimensional coordinate; δ, wall thickness; $\xi = x/\delta$, dimensionless coordinate; f, crosssectional area of metal skeleton per m^2; h, distance between centers of openings; P, wetted perimeter of channels per m^2 of cross section; $Bi_\delta = \alpha P \delta^2/\lambda f$, Biot number; $g = \rho(1 - f)uc_p\delta/\lambda f$, dimensionless fluid flow rate; t_M, dimensional metal temperature; t_F, dimensional fluid temperature; t_{0M}, t_{1M}, t_{0F}, initial temperature of metal and surfaces and inlet temperature of fluid.

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REGENERATIVE HEAT EXCHANGERS WITH PERIODIC TIME VARIATION OF COOLANT TEMPERATURE

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Analytic expressions are derived for the temperatures of a heat-exchanger solid and coolant.

Operating elements of power plants and chemical reactors are frequently subjected to the action of a coolant whose temperature varies periodically with time. The change in temperature of the solid parts of these elements determines the service characteristics of the plant or reactor, e.g., the change in the rate of catalytic reactions, etc. From the point of view of heat transfer such installations are regenerative heat exchangers.

Let us consider a heat exchanger whose heat-retaining elements have such characteristics that the temperature gradient in elements of the solid can be assumed negligibly small. In addition, we assume that the gas or liquid (from now on for brevity we refer only to a gas) moves through the free space of the heat exchanger of length L in such a way that the temperature of the gas or solid is the same at all points of a cross section of the heat exchanger perpendicular to the direction of gas flow.

Then, following [1], the problem is reduced to that of solving the following system of equations:

$$\frac{\partial T_1}{\partial t} = a(T_2 - T_1), \tag{1}$$

$$\frac{\partial T_2}{\partial x} + \theta \frac{\partial T_2}{\partial t} = b(T_1 - T_2), \tag{2}$$

where $a = \alpha A l/c_1 M l$; $b = \alpha A l/c_2 \dot{m}$; $\theta = \rho_2 V l/\dot{m}$; $0 \leqslant t > \infty$.

The boundary and initial conditions follow from the formulation of the problem:

$$T_1(x,\ t)|_{t=0} = 0,\ T_2(x,\ t)|_{t=0} = 0, \tag{3}$$

$$T_2(x,\ t)|_{x=0} = D \sin \omega t. \tag{4}$$

In most treatments of regenerative heat exchangers the second term on the left-hand side of Eq. (2) is assumed negligibly small in comparison with the first. To obtain a more general solution we retain this term in the initial system of equations.

After taking Laplace transforms of Eqs. (1)-(4) we have

$$p\bar{T}_1 = a(\bar{T}_2 - \bar{T}_1), \tag{5}$$

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